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Optimum Profile Shift Estimation on Direct Design Asymmetric Normal and High Contact Ratio Spur Gears Based on Load Sharing

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Abstract

In direct design method, gear pair is selected from area of existence diagram. This area of existence diagram gives N number of possible gear pairs. From this possible gear pairs it is important to optimize the fillet strength of pinion and gear with respect to the optimum profile shift. Optimum design of gear drive means the maximum fillet stresses developed at the pinion and gear are to be equal. Hence the present work attempts for a reasonably accurate estimation of optimum profile shifts based on pinion and gear tooth fillet strength for a load at critical loading point considering the load sharing between the gear pair on asymmetric normal and high contact ratio asymmetric spur gear through direct design.

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Keywords: Asymmetric Spur Gear, Direct Design, Normal Contact Ratio, High Contact Ratio, Profile Shift, Fillet Stress.

1. Introduction

The load carrying capacity of gear drive is enhanced by different methods such as heat treatment process (i.e. case hardening and shot peening), modification of gear geometry as asymmetric gears with high contact ratio and direct design procedures. Conventional gear design and direct gear design are the two major approaches used to design gears of which conventional gear design is based on standard rack cutter and the direct gear design [1] is an application driven gear development process without concern for any predefined tooling parameters. Direct gear design defines the gear tooth details without using the generating rack parameters like module, addendum height and pressure angle. Tool characteristics and manufacturing operations are secondary. In general, the profile on one side of a gear tooth is functionally different from that of the other side. The work load on the profile of one side is

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significantly higher and is applied for a longer period than at the other side. The design intent is to improve the performance of the primary contacting profile by degrading the performance of the opposite side profile. The applications of asymmetric gears are wind mill, Helicopter main gearbox and turbo-prop engine drives.

The direct design procedure is developed to increase the load capacity of gears with reduced weight. Gang Deng et al. [2] estimated load sharing ratio (LSR) based on bending stiffness for different standard pressure angle on coast side and high pressure angle on drive side. Muni et al. [3] optimized the fillet strength for direct design asymmetric normal contact ratio (NCR) spur gear for a load at highest point of single tooth contact (HPSTC). Senthil Kumar et al. [4] developed a non-standard asymmetric rack cutter to generate asymmetric gear and optimized the fillet strength with optimum profile shift. Rama Thirumurugan and Muthuveerappan [5, 6] developed a finite element model for a reasonably accurate estimate of the fillet stress based on load sharing for symmetric NCR spur gears and they have suggested that multi pair contact model (MPCM) yields reasonably accurate result than single point load model and found critical loading points for symmetric NCR and symmetric High Contact Ratio (HCR) gears based on load sharing. In addition to that they found influence of profile shift on symmetric NCR spur gear based on load sharing. Baglioni et al. [7] investigated the effect of profile shift over the efficiency and suggested that increase in sum of addendum modification results to increase the gear efficiency.

It is found that very limited literatures are available to find the optimum profile shift on asymmetric NCR and HCR gear drives through direct design. Hence, the present work is attempted for a reasonably accurate estimation of optimum profile shift considering the LSR based fillet stress.

2. Direct Gear Design

2.1. Area of Existence for Asymmetric Normal and High Contact Ratio Spur Gear

The area of existence diagrams on asymmetric NCR and HCR gear drives developed for some given gear parameters respectively are shown in Fig. 1(a) and 1(b). Each diagram gives a number of possible solutions of asymmetric gear pair. Fig. 1(a) consists of four isograms for known input values i) number of teeth in pinion (z_1) and gear (z_2), ii) top land thickness coefficients pinion (m_{a1}) and gear (m_{a2}), iii) coefficient of asymmetry (k) and iv) drive side contact ratio (ε_d). The isogram A_i is a line for constant contact ratio on drive side, which is found using eq. (3). From the area of existence, the possible existence of design solutions, for the drive of some given input parameters at different addendum pressure angles (α_{ad1}-α_{ad2}) of the pinion and the gear are made to develop the rack dimensions for the required constant drive side contact ratio. The isograms B_i are drawn for the pinion and the gear such that B_1 for α_{lc1}=0 and B_2 for α_{lc2}=0. The isograms C_i are drawn for the pinion and gear such that C_1 for $\alpha_{ad1} \geq \alpha_{od}$, and C_2 for $\alpha_{ad2} \geq \alpha_{od.}$ This area is built in (α_{ad1}-α_{ad2}) coordinates and includes the number of isograms reflecting the constant values of different gear parameters. The isograms A_i, B_i, and C_i have been accomplished from the minimum requirements of gear mesh conditions such that

1. The isograms A_i represent the drive side contact ratio (ε_d). For continuous contact, it should be greater than or equal to one (i.e. $\varepsilon_d \geq 1$).

2. The isograms B_i are representing the interference free condition. To avoid the interference at the fillet the pressure angle at the limiting circles of pinion and gear should be greater than or equal to zero (i.e. B_1 for $\alpha_{lc1} \geq 0$ and $\alpha_{lc2} \geq 0$).

3. The isograms C_i are drawn based on minimum addendum condition. To meet this the addendum pressure angle of pinion or gear should be greater than drive side pressure angle at pitch point (i.e. $\alpha_{ad1} \geq \alpha_d$ and $\alpha_{ad2} \geq \alpha_d$).

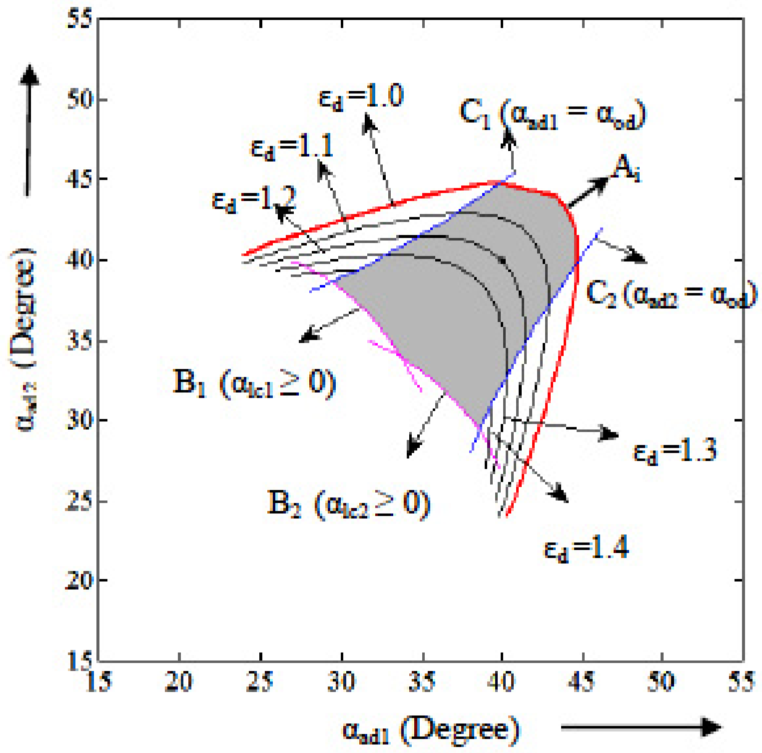


Fig. 1(a). Area of existence diagram for NCR gear

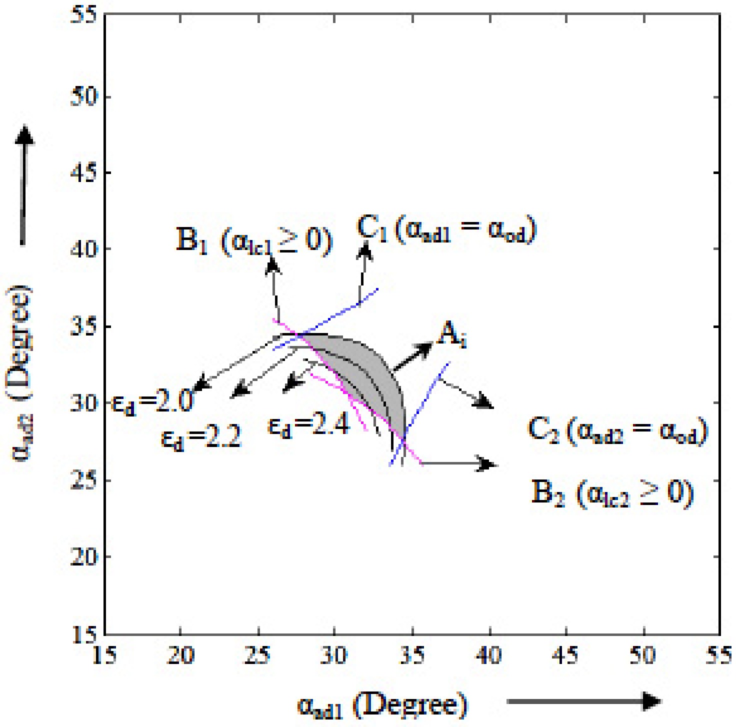


Fig. 1(b). Area of existence diagram for HCR gear

The coefficient of asymmetry (k) and top land thickness coefficient (m_a), α_{od}, α_{oc}, ε_d, ε_c, α_{lc1} and α_{lc2} are defined by Kapelevich, 2000 are given in the eqs. (1) – (7).

$$k = \frac{d_{bc}}{d_{bd}} = \frac{\cos v_c}{\cos v_d} = \frac{\cos \alpha_{bc}}{\cos \alpha_{bd}} = \frac{\cos \alpha_{oc}}{\cos \alpha_{od}}, \tag{1}$$

$$m_a = \frac{invv_d + invv_c - inv\alpha_{ad} - inv\alpha_{ac}}{2 * \cos \alpha_{ad}} \tag{2}$$

$$inv\alpha_{od} + inv\alpha_{oc} = \frac{invv_{1d} + invv_{1c} + i*(invv_{2d} + invv_{2c}) - \frac{2\pi}{z_1}}{i+1} \tag{3}$$

$$\varepsilon_d = z_1 * \left(\frac{\tan \alpha_{ad1} + i * \tan \alpha_{ad2} - (1+i) * \tan \alpha_{od}}{2\pi} \right) \tag{4}$$

$$\varepsilon_c = z_1 * \left(\frac{\tan \alpha_{ac1} + i * \tan \alpha_{ac2} - (1+i) * \tan \alpha_{oc}}{2\pi} \right) \tag{5}$$

$$\tan \alpha_{lc1} = (1+i) * \tan \alpha_{oc} - i * \tan \alpha_{ac2} \geq 0 \tag{6}$$

$$\tan \alpha_{lc2} = \frac{(1+i) * \tan \alpha_{oc}}{i} - \frac{\tan \alpha_{ac1}}{i} \geq 0 \tag{7}$$

A non-standard asymmetric rack cutter is developed to generate the direct design asymmetric gear profiles. The relevant cutter parameters such as cutter tooth thickness, profile shift (x) and cutter tip radius (A) are given by the eqs. (8) to (11), (Kapelevich, 2000).

$$T_w = r_w * (invv_d + invv_c - inv\alpha_{wd} - inv\alpha_{wc}) \tag{8}$$

$$T_{wr} = \pi m_r - T_w \tag{9}$$

$$x = \frac{\frac{\pi m_r}{2} - T_{wr}}{\tan \alpha_{wd} + \tan \alpha_{wc}} \tag{10}$$

$$A = \frac{(\tan \alpha_{wd} + \tan \alpha_{wc}) * (r_w - r_f + x) - \frac{\pi m_r}{2}}{(\tan \alpha_{wd} + \tan \alpha_{wc}) - \frac{1}{\cos \alpha_{wd}} - \frac{1}{\cos \alpha_{wc}}} \tag{11}$$

3. Multi Pair Contact Analysis

In this present work MPCM is used to estimate LSR, LSR based fillet stress and contact stress using the two-dimensional (2D) three teeth full rim model for NCR (Fig.2(a)) and 2D five teeth full rim model for HCR gear drives (Fig. 2(b)). The Ansys Parametric Design Language code (ANSYS 12.1) was developed to generate 2D three teeth full rim model. In this study plain strain condition is assumed. An eight nodded quadrilateral (in ANSYS, 2D-PLANE 82) with two degrees of freedom at each node is used to get Finite Element (FE) model of asymmetric NCR spur gear. The contact element (CONTA172) and target element (TARG169) are used to establish surface to surface contact between pinion and gear. The inner periphery of the gear is radially restrained in all direction and normal force (F_n =10N) with equivalent torque applied at inner periphery of the pinion. The Augmented Legrangian contact algorithm is chosen by default. A convergence study is carried out to fix the element size at the fillet and contact region. The maximum fillet stress ($(\sigma_t)_{max}$) for a load at critical loading point namely the HPSTC for NCR spur gear is obtained at root and its position with respect to optimum profile shift is shown in Fig. 3. The LSR is defined as the ratio between the contact forces at a pair (F_{ni}) to total normal load is given by the eq. (12).

$$LSR = \frac{F_{ni}}{F_n} \tag{12}$$

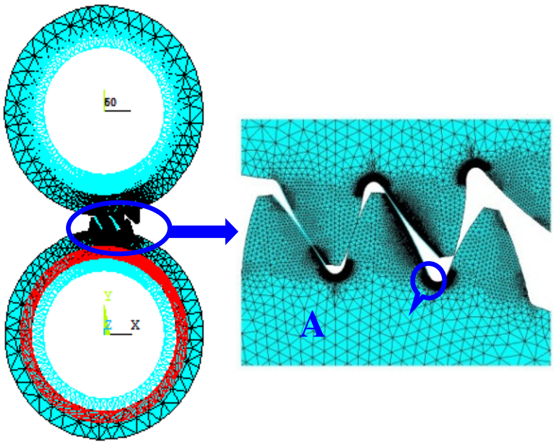


Fig. 2(a). 2D Three teeth full rim model of NCR gear

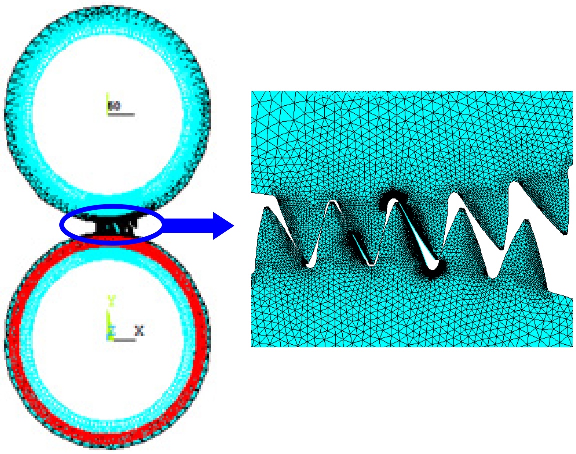


Fig. 2(b). 2D Five teeth full rim model of HCR gear

Table.1 NCR and HCR gear parameters for optimum profile shift

Input Gear Parameters	Value	
	NCR	HCR
Number of teeth in the pinion(z_1)	40	50
Gear ratio (i)	1.0	2.0
Coefficient of asymmetry (k)	1.1	1.1
Top land thickness coefficient m_a)	0.4/z	0.25/z
Contact ratio-drive side (ε_d)	1.40	2.10
Module	1.0 mm	1.0 mm
Generated Asymmetric Rack Cutter Parameters		
Pressure angle(Drive side)-α_{rd}	33.24°	28.77°
Pressure angle(Coast side) -α_{rc}	23.07°	15.37°
Tooth thickness (t_r)	1.5618	1.561
Tip Radius (A_r)	0.3 mm	0.2 mm

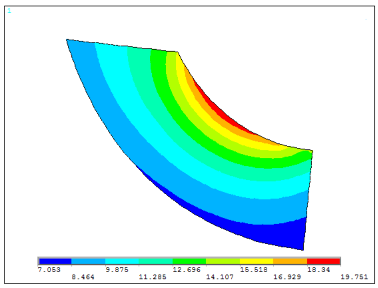
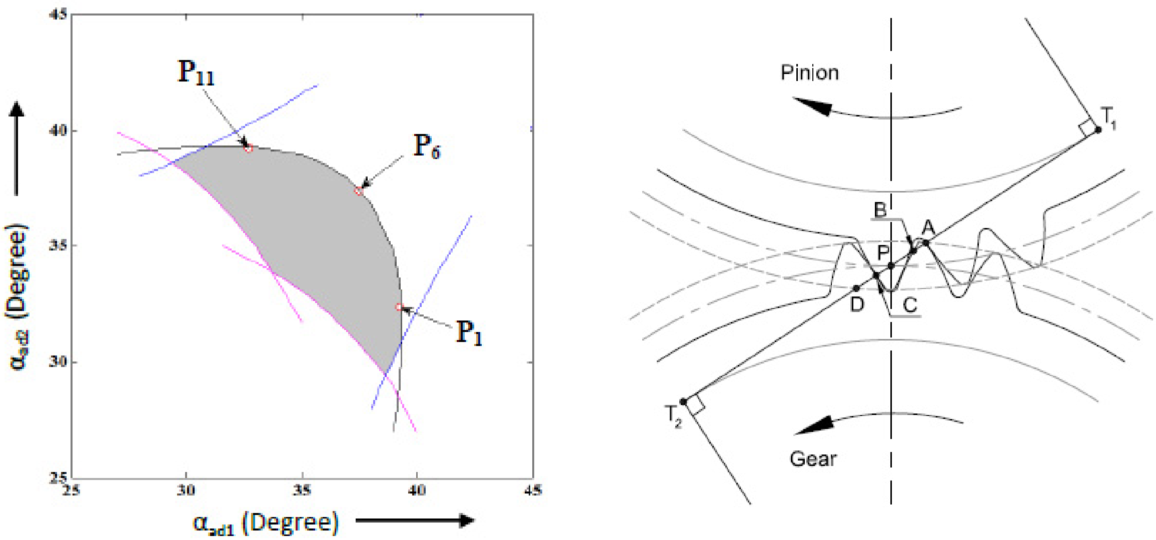


Fig. 3 Maximum Fillet stress plot for asymmetric NCR spur gear for a load at HPSTC (Magnification of A in Fig. 2(a)).

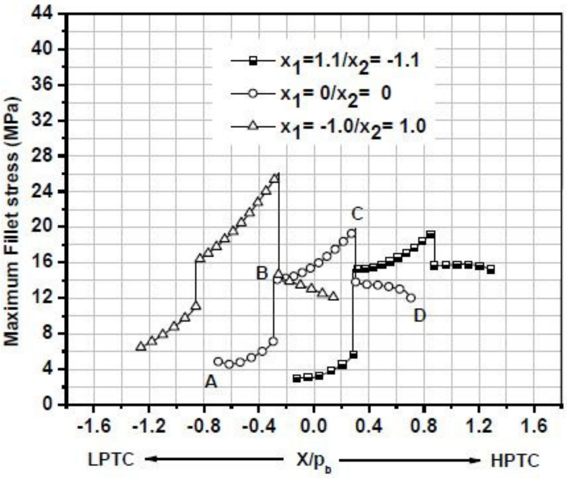
Material Properties		
Young’s Modulus (E)	210 GPa	210 GPa
Poisons Ratio (γ)	0.3	0.3

4. Optimum Profile Shifts on Asymmetric Normal Contact Ratio Spur Gear

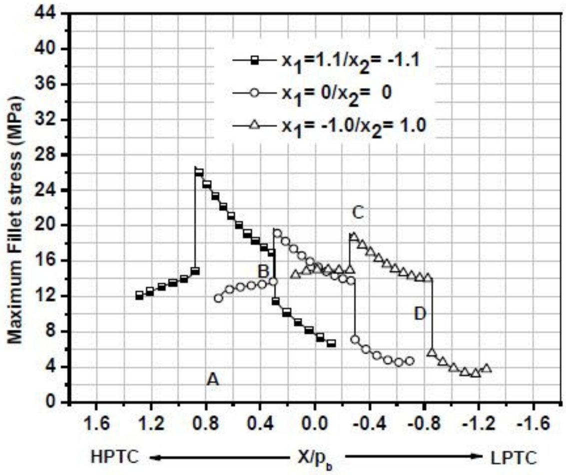
The area of existence diagram for known input values m=1, z_1=40, i=1, k=1.1, ε_d=1.4 is shown in Fig. 4(a). This area of existence diagram gives N number of possible gear pairs for a given ε_d. From this possible gear pairs, it is important to balance the fillet strength of pinion and gear with respect to the optimum profile shift. For example a particular contact ratio (ε_d=1.4), several points P_1 to P_{11} are selected from the area of existence diagram and corresponding rack parameters for pinion and gear are determined, from which the respective gear pairs are generated. The point P_1 (α_{ad1}=39.2488, α_{ad2}=32.3709) in this diagram generates a rack cutter parameters with positive profile shift of pinion (x_1=1.1) and negative profile shift on gear (x_2= -1.1). Similarly the point P_{11} (α_{ad1}=32.6878, α_{ad2}=39.2410) generates the rack cutter with x_1= -1.0 and x_2= 1.0 and the point P_6 (α_{ad1}=37.4595, α_{ad2}=37.3988) generates cutter parameters with zero (x_1=0, x_2 = 0) profile shifts. The Fig. 4(b) shows the meshing of pinion and gear for profile shift. The maximum fillet stresses at the pinions and gears against the contact position for points P_1, P_6 and P_{11} are shown in Figs. 4(c) and 4(d). The LSR based maximum fillet stress at the pinions and gears of eleven sets and corresponding to the points P_1 to P_{11} are determined through MPCM and plotted as shown in Fig. 4(e). The intersecting point of the curves is considered as the optimum point and it is inferred that the maximum fillet stress at the pinions and gears are balanced for the drive developed through the point P_6 in the isogram. This point is considered as optimum point and the respective gear and rack parameters are shown in Table.1



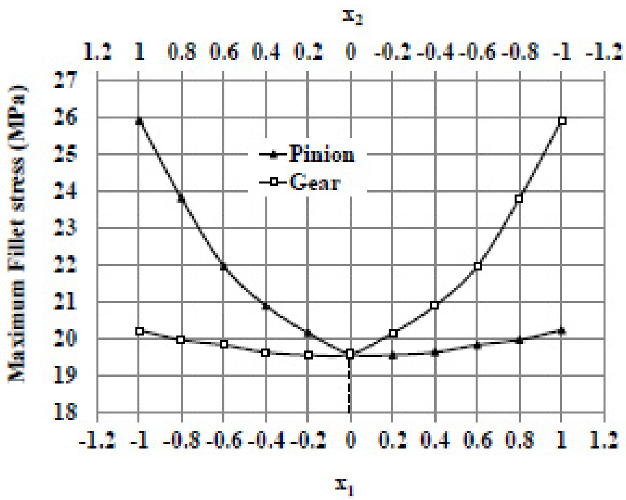
(a) (b)



(c)



(d)

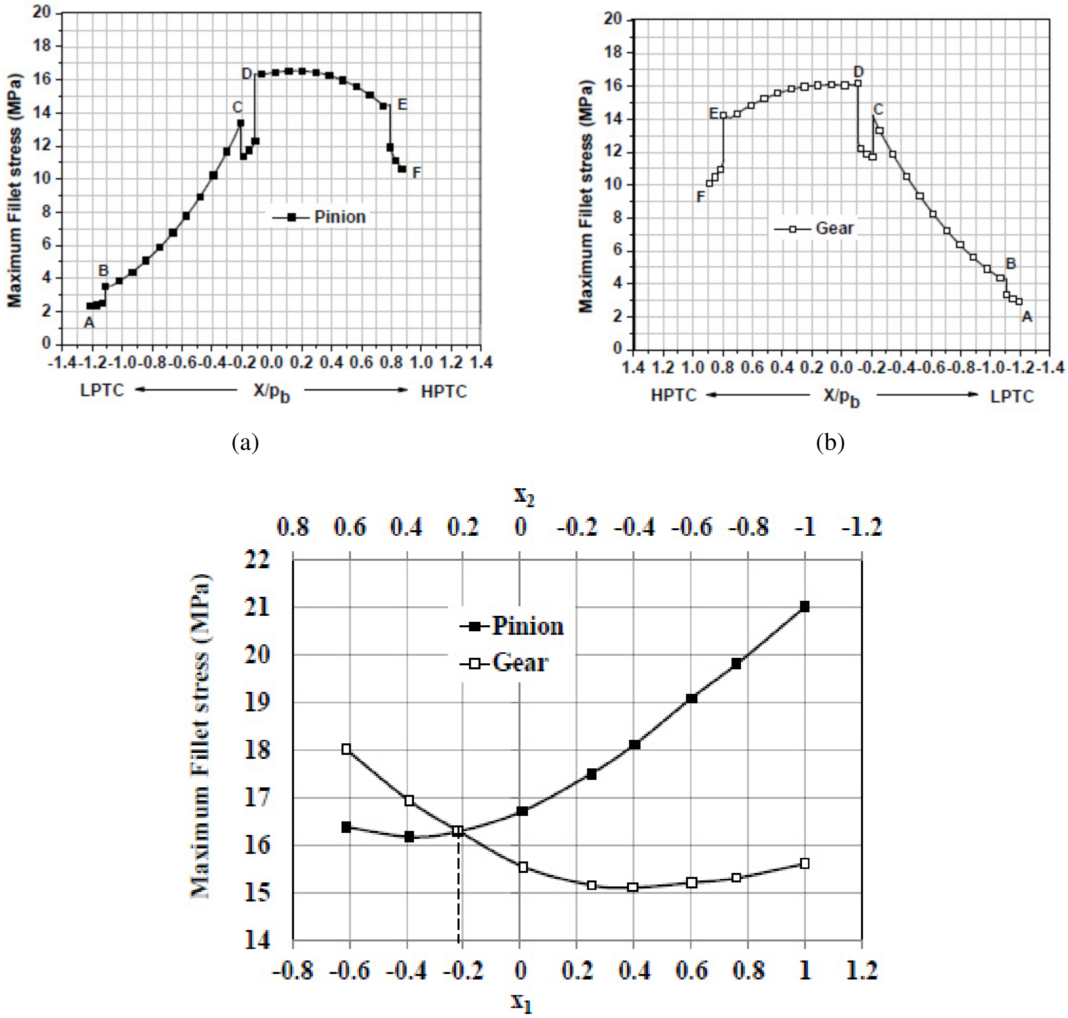


(e)

Fig. 4 Optimum profile shifts estimation based on $(\sigma_t)_{max}$ for a load at high pressure angle side (m=1, z_1=40, i=1, k=1.1, ε_d=1.4). a) Area of existence diagram. b) Meshing of pinion and gear for optimum profile shift. c) $(\sigma_t)_{max}$ vs. contact position for pinion. d) $(\sigma_t)_{max}$ vs. contact position for Gear. e). $(\sigma_t)_{max}$ vs. profile shift on pinion and gear.

5. Optimum Profile Shifts Of The Pinion And Gear For An Asymmetric High Contact Ratio Spur Gear Drives.

The critical loading point of an asymmetric NCR gear drives is always HPSTC only. But, the critical loading point of asymmetric HCR gear drives changes with respect to gear parameters such as drive side pressure angle, profile shift, module, teeth number and addendum height. In general the critical loading point for $(\sigma_t)_{max}$ is always lies between second lowest point of double tooth contact (SLPDTC- denoted as D) [6] and second highest point of double tooth contact (SHPDTC- denoted as E) [6]. In the present work, as described in the NCR gear drive, nine points are selected in the isogram for a particular ε_d=2.1 and the respective rack cutter sets are generated. The $(\sigma_t)_{max}$ of pinion and gear for optimum profile shift is shown in Fig. 5(a) and 5(b). The plot for the $(\sigma_t)_{max}$ versus profile shifts of pinion and gear for all the sets of drives are shown in Fig. 5(c). The optimum profile shifts are identified at the intersecting point of the curve are they x_1= -0.21 and x_2=0.21 as observed from the Fig. 5(c).



(a) (b)

(c)

Fig. 5 Optimum profile shifts estimation based on $(\sigma_t)_{max}$ for a load at high pressure angle side (m=1, z_1=50, i=2, k=1.1, ε_d=2.1). a) $(\sigma_t)_{max}$ vs. contact position for pinion. b) $(\sigma_t)_{max}$ vs. contact position for Gear e). $(\sigma_t)_{max}$ vs. profile shift on pinion and gear.

6. Conclusions

This present work attempted to find a reasonably accurate estimation of optimum profile shifts for asymmetric NCR and HCR gear drives considering the LSR based maximum fillet stress through direct design for the known input gear drive parameters. The following results are obtained from this work.

1. The area of existence diagrams for asymmetric HCR gear is developed.

2. The critical loading point for asymmetric HCR gear is found and the respective stresses are considered for optimization.

3. It is found that the optimum profile shift of asymmetric HCR gear for i=2 is x_1= -0.21 and x_2= 0.21. The $(\sigma_t)_{max}$ for pinion = $(\sigma_t)_{max}$ for gear = 16.27 MPa for a normal force 10N.

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